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THE ROLLING AND SLIPPING OF A CYLINDER ALONG THE BOUNDARY OF AN IDEALLY PLASTIC HALF-SPACE†

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A solution of the problem of steady plane plastic flow when a smooth or rough circular cylinder rolls and slips along the boundary of an ideally plastic half-space is obtained. Cases of forward and backward slipping of the material along the contact surface with a change in the direction of contact friction are examined. Limiting values of the arc of contact and the forces acting on the cylinder for plastic flow is possible are obtained. © 2003 Elsevier Science Ltd. All rights reserved.

Problems of the rolling of a rigid cylinder along a viscoelastic half-space [1, 2] and the rolling of a smooth cylinder along a rigid plastic half-space [3] were considered earlier; in the latter case, the solution was obtained using the small-parameter method for an arc of contact that is small compared with the radius of the cylinder. A review of the most important investigations of the problem of rolling friction was also given in [1–3].

The rolling of a cylinder without slipping along the boundary of an elastic plastic half-space was analysed using the finite-element method in [4, 5]. However, the large deformations which occur in the plastic contact zone for large loads on the cylinder, the problem of the unknown stationary boundary of the plastic region, and the singularity of the stress and displacement velocity fields at the point of intersection of the free plastic boundary with the cylinder make elastic-plastic modelling of the rolling and slipping of the cylinder considerably more difficult.

The problem of the rolling and slipping smooth and rough rigid cylinders is solved below for an ideally plastic model of a half-space, using as a basis the hyperbolic equations of plane strain [6]. Steady plastic flow with the formation of a curved free boundary in front of the cylinder is examined.

It is shown that the determination of this boundary and of the whole plastic region leads to the solution of a non-linear vector equation relating to pressure distribution on the contact boundary. Limiting values at which steady plastic flow is possible are obtained for the arc of contact and loading of the cylinder. For a rough cylinder, two versions of the problem are examined, with a change in the direction of the contact friction as a function of the direction of slip of the plastic material along the boundary of contact with the cylinder. The relation between this problem. The problem of rolling friction, and the rolling and drawing of thick blanks with plastic deformation of the surface layer is discussed.

1. FORMULATION OF THE PROBLEM AND BASIC EQUATIONS

Figure 1 shows the shape of the plastic region during the rolling and slipping of a rigid circular cylinder along the boundary of an incompressible ideally plastic half-space [3]. The axis of the cylinder is considered to be stationary, but the non-deformed half-space is considered to be moving at unit velocity $V = 1$. Owing to plastic incompressibility, the points O and B are on the boundary $y = 0$. Free from external stresses, the boundary AB coincides with the line of flow, since the process of plastic flow is stationary.

If the cylinder is smooth, the shape and size of the plastic region depend solely on the vertical force Q and the horizontal force F applied to the axis, and do not depend on the rotation of the cylinder, since the boundary conditions at the contact boundary OA are the same whether or not the cylinder is rotating. It the cylinder is rough, then during slip of the plastic material from point O towards point A, shear stresses of contact friction appear, creating a positing moment M. This is the case of the rolling of a cylinder without slipping at the point of contact with the rigid region and with forward slip of

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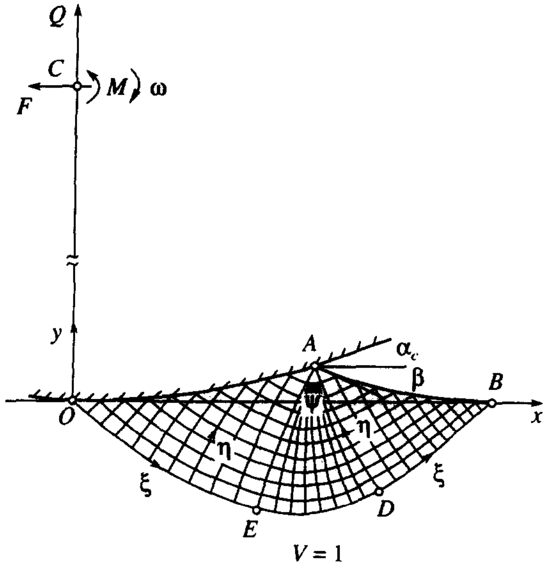

Fig. 1

the material on the arc of contact. When the material slips along the surface of a rough cylinder in the opposite direction, The contact friction stresses and moment M change signs. This is the case of the slipping of a cylinder without rotation or of the slipping and rotation of a cylinder with a low angular velocity ω a backward slipping on the entire arc of contact.

We will take as the unit of stress double the shear yield stress of the material and we will take as the unit of length the arc of contact OA. For plane plastic flow, the differential equations of the ξ and η slip lines and Hencky's relations for the stresses and Geiringer's relations for the displacement velocities along the slip lines take the form

$$dy/dx = \operatorname{tg}\varphi \quad \text{for} \quad \xi, \quad dy/dx = -\operatorname{ctg}\varphi \quad \text{for} \quad \eta \tag{1.1}$$

$$d\sigma - d\varphi = 0 \quad \text{along} \quad \xi, \quad d\sigma + d\varphi = 0 \quad \text{along} \quad \eta \tag{1.2}$$

$$dV_\xi - V_\eta d\varphi = 0 \quad \text{along} \quad \xi, \quad dV_\eta + V_\xi d\varphi = 0 \quad \text{along} \quad \eta \tag{1.3}$$

where σ is the mean stress, φ is the slope of the tangent to the ξ slip line with the x axis, and V_ξ and V_η are projections of the velocity vector onto ξ and η.

Because the plastic flow is steady, the boundary AB, free from external stresses, coincides with the streamline of flow, i.e.

$$\operatorname{tg}(\varphi - \pi/4) = V_y/V_x, \quad \sigma = -\tfrac{1}{2} \quad \text{on} \quad AB \tag{1.4}$$

where V_x and V_y are projections of the velocity vector onto the axes of the coordinates x and y, related to V_ξ and V_η by the equations

$$V_x = V_\xi \cos\varphi - V_\eta \sin\varphi, \quad V_y = V_\xi \sin\varphi + V_\eta \cos\varphi \tag{1.5}$$

Along the rigid plastic boundary $OEDB$ the velocities are constant: $V_x = -1$, $V_y = 0$. From Eqs (1.5) we obtain

$$V_\xi = -\cos\varphi, \quad V_\eta = \sin\varphi \quad \text{on} \quad OEDB \tag{1.6}$$

Since the arc of contact OA is taken to be a characteristic dimension, the radius of the cylinder R and the contact angle α_c are related by the equation $R\alpha_c = 1$. The velocity of the surface of the cylinder can vary in the range $0 \leqslant \omega R \leqslant 1$. During rolling without slipping we have $\omega R = 1$ at the point O.

If the cylinder is smooth, then the slip lines intersect the boundary OA at an angle of $\pi/4$. Hence we find

$$\varphi = \alpha - \pi/4, \quad 0 \le \alpha \le \alpha_c \quad \text{on} \quad OA \tag{1.7}$$

In the case of a rough cylinder, shear stresses of contact friction τ_c arise on the boundary OA, and the slip lines intersect this boundary at an angle

$$\theta = \tfrac{1}{2} \arccos 2\tau_c, \quad 0 \le \tau_c \le \tfrac{1}{2} \tag{1.8}$$

If $\omega R > V_c$, where V_c is the velocity of the material at the contact boundary, then forward slipping occurs, and an angle θ is created between the η slip line and the tangent to OA. If $\omega R < V_c$ then backward slipping occurs with a change in the direction of τ_c, and an angle θ is created between the ξ slip line and the tangent to OA. Therefore, the angle φ depends on the direction in which the material slips relative to the cylinder surface

$$\varphi = \alpha + \theta - \pi/2, \quad \text{if} \quad \omega R > V_c; \quad \varphi = \alpha - \theta, \quad \text{if} \quad \omega R < V_c \tag{1.9}$$

Since the velocity normal to the cylinder is zero, the kinematic boundary conditions on OA are: for a smooth cylinder

$$V_\xi = V_\eta \tag{1.10}$$

for a rough cylinder

$$V_\xi = V_\eta \operatorname{tg}\theta, \quad V_\xi = V_\eta \operatorname{ctg}\theta \tag{1.11}$$

during forward and backward slipping respectively.

The slip line field and the displacement velocity field corresponding to it can only be plotted for positive values of the angle ψ of a fan centred at point A, which is defined by the following expressions: for a smooth cylinder

$$\psi = \pi/2 - (a_c + \beta) \tag{1.12}$$

for a rough cylinder

$$\begin{aligned} \psi &= 3\pi/4 - (a_c + \beta + \theta), \quad \text{if} \quad \omega R > V_c \\ \psi &= \pi/4 + \theta - (a_c + \beta), \quad \text{if} \quad \omega R < V_c \end{aligned} \tag{1.13}$$

Here

$$\beta = -\operatorname{arctg}(V_y/V_x)_A \tag{1.14}$$

is the slope of the tangent to the boundary AB at the point A.

The mean stress at point O can be found from Hencky's equation for the ξ slip line $OEDB$ and boundary conditions (1.7)–(1.9) when $\alpha = 0$.

For a smooth cylinder

$$\sigma_0 = -\tfrac{1}{2}(1 + \pi) \tag{1.15}$$

With this value of σ_0 a rigid wedge with its apex at the point O is loaded to the limiting plastic state, since the boundary of the half-space to the left of the point O is stress-free. For a rough cylinder for forward slipping

$$\sigma_0 = -\tfrac{1}{2}(1 + 3\pi/2) + \theta \tag{1.16}$$

and for backward slipping

$$\sigma_0 = -\tfrac{1}{2}(1 + \pi/2) - \theta \tag{1.17}$$

The angle θ is given by Eq. (1.8). Expression (1.16) shows that, during forward slipping, the load-carrying capacity criterion of the rigid wedge at point O is satisfied when $\tau_c = 0$ ($\theta = \pi/4$) at that point; it follows that, in such a case the plastic region shown in Fig. 1 can only be plotted for a variable contact friction stress with a zero value at the point O. During backward slipping, the load-carrying capacity criterion of the rigid wedge at point O is satisfied for all values of τ_c.

The rolling of a cylinder with forward slipping corresponds to the limiting case of the rolling of thick workpieces, in which plastic deformation only occurs in the surface layer, and the neutral section passes through the point O. In this case, in the theory of rolling [7] based on experimental data it is assumed that there is a linear variation in the contact friction stresses along the arc of contact with a zero value in the neutral section. For the rolling of a cylinder with forward slipping we assume a linear variation of τ_c from zero at the point O to a maximum value at the point A, where the slipping velocity relative the cylinder surface is greatest. This agrees qualitatively with Nadai's model in which the viscous resistance to shear of the contact layer is proportional to the slipping velocity [7].

When a cylinder rolls with backward slip, slipping over the cylinder surface occurs along the entire arc of contact. The limiting case of this problem is the slipping of the cylinder without rotation, when the slipping velocity of the material over the cylinder surface varies negligibly along the small arc of contact. During backward slipping the value of τ_c is assumed to be constant over the entire arc of contact. The condition τ_c = const with high contact pressures agrees more closely with experiments than Coulomb's law of dry friction [7], and in technological problems of the theory of plasticity it is usually known as Prandtl's law. Here τ_c is considered to be the shear strength of the material of the contact layer, which depends on the lubricant and the state of the contact surface and does not exceed the shear yield stress of the material of the main plastic region.

The mean stress σ decreases in absolute value along the arc of contact OA, and at the point A takes a value which depends on the angle ψ

$$\sigma_A = -\tfrac{1}{2}(1+2\psi) \tag{1.18}$$

If the distribution of σ along the arc of contact OA is known, then the normal pressure on the cylinder can be determined

$$-\sigma_n = -(\sigma - \tfrac{1}{2}\sin 2\theta) \tag{1.19}$$

and it is possible to find the forces and torque, which, if the relation $R\alpha_c = 1$ is taken into account, takes the form

$$\begin{gathered} Q = \frac{1}{a_c}\int_0^{\alpha_c}[(-\sigma_n)\cos\alpha \pm \tau_c\sin\alpha]d\alpha, \quad F = \frac{1}{\alpha_c}\int_0^{\alpha_c}[(-\sigma_n)\sin\alpha \mp \tau_c\cos\alpha]d\alpha \\ M = \pm\frac{1}{\alpha_c^2}\int_0^{\alpha_c}\tau_c d\alpha \end{gathered} \tag{1.20}$$

The upper signs relate to forward slipping, and the lower signs to backward slipping.

For a smooth cylinder

$$-\sigma_n = -(\sigma - \tfrac{1}{2}), \quad M = \tau_c = 0$$

For a rough cylinder with forward slipping and a linear variation in τ_c, from the above expression (1.20) we find $M = \tfrac{1}{2}(\tau_c)_A/\alpha_c$, and with backward slipping $M = -\tau_c/\alpha_c$.

Conditions (1.4) and (1.12)–(1.14) show that the problem of the rolling and slipping of a cylinder needs to be studied in conjunction with the stress and displacement velocity fields. The governing equation for solving the problem can be found as follows. We will specify a constant distribution of the mean stress σ on the boundary OA, taking into account the known values (1.15)–(1.17) at the point O and the initial approximation for the angle β. Then the values of σ and the boundary conditions for the angle φ (1.7)–(1.9) determine the Cauchy data for Eqs (1.1) and (1.2) and enable the field of slip lines to be found in the region OAE. In the region AED the slip line field can be found by solving Goursat's problem with given σ and φ on AE and at the singular point A with a known angle ψ (1.12)–(1.14). In the region ABD the inverse Cauchy problem is solved with given σ and φ on AD and with the condition $\sigma = -\tfrac{1}{2}$ and $dy/dx = \mathrm{tg}(\varphi - \pi/4)$ on the unknown boundary AB. As a result we find the free boundary AB and the rigid plastic boundary $OEDB$. We then determine the displacement velocity field in the plastic region from the solution of the mixed boundary-value problem for Eqs (1.3) with boundary conditions (1.6), (1.10), and (1.11). If the steady flow condition (1.4) is satisfied on the boundary AB, then the distribution of σ on OA is the solution to the problem. Since the procedure for solving boundary-value problems for Eqs (1.1)–(1.3) determines the slip line field with the boundary AB and the displacement velocity field as a function of the distribution of σ on OA, then condition (1.4) is the governing equation for the unknown distribution of σ on OA, which can be described in the operator form

$$\Phi(\sigma) \equiv \operatorname{tg}(\varphi - \pi/4) - V_y/V_x = 0 \quad \text{on} \quad AB \tag{1.21}$$

where Φ is the algorithm for calculating the boundary AB and the displacement velocities of the points of this boundary, which is implemented here in the form of numerical procedures. In this case, Eq. (1.21) is a non-linear, finite-dimensional vector equation, which is solved by Broyden's iteration method [8] for determining the values of σ at nodes on the boundary OA.

2. NUMERICAL SOLUTION OF THE PROBLEM

The calculation of the slip line field and the displacement velocity field with a given distribution of stress σ on the boundary OA is based on a finite-difference approximation of differential equation (1.1)–(1.3). In the regular region elementary Cauchy problems are solved with known values of σ, φ, V_ξ, and V_η at points 1 and 2 on the slip lines ξ and η in the vicinity of an unknown point P (Fig. 2a). From Eqs (1.2) we find σ and φ at the point P

$$\sigma = {}^1\!/\!_2\,(\sigma_1 + \sigma_2 - \varphi_1 + \varphi_2), \quad \varphi = {}^1\!/\!_2\,(\varphi_1 + \varphi_2 - \sigma_1 + \sigma_2) \tag{2.1}$$

where the subscripts 1 and 2 relate to the values of the variables at points 1 and 2. Then, from Eqs (1.1) we find the coordinates of the point P for the mean values of the angle φ between points 1 and $P\langle\varphi_1\rangle = {}^1\!/\!_2(\varphi_1 + \varphi)$ and between points 2 and $P\langle\varphi_2\rangle = {}^1\!/\!_2(\varphi_2 + \varphi)$:

for $\langle\varphi_1\rangle \neq 0$ and $\langle\varphi_2\rangle \neq 0$

$$\begin{aligned} x &= [y_2 - y_1 + x_1 \operatorname{tg}\langle\varphi_1\rangle + x_2 \operatorname{ctg}\langle\varphi_2\rangle]/(\operatorname{tg}\langle\varphi_1\rangle + \operatorname{ctg}\langle\varphi_2\rangle) \\ y &= [x_2 - x_1 + y_1 \operatorname{ctg}\langle\varphi_1\rangle + y_2 \operatorname{tg}\langle\varphi_2\rangle]/(\operatorname{tg}\langle\varphi_2\rangle + \operatorname{ctg}\langle\varphi_1\rangle) \end{aligned} \tag{2.2}$$

for $\langle\varphi_1\rangle = 0$ or $\langle\varphi_2\rangle = 0$

$$x = x_2, \quad y = y_1$$

The velocities V_ξ and V_η are found from Eqs (1.3) for the known value of the angle φ at the point P

$$\begin{aligned} V_\xi &= [V_{\xi 1} + (V_{\eta 1} + V_{\eta 2})a_1 - V_{\xi 2}a_1 a_2]/(1 + a_1 a_2) \\ V_\eta &= [V_{\eta 2} - (V_{\xi 1} + V_{\xi 2})a_2 - V_{\eta 1}a_1 a_2]/(1 + a_1 a_2) \\ a_1 &= {}^1\!/\!_2\,(\varphi - \varphi_1), \quad a_2 = {}^1\!/\!_2\,(\varphi - \varphi_2) \end{aligned} \tag{2.3}$$

where the subscripts 1 and 2 relate to the values of the velocities at points 1 and 2.

The stress-free boundary AB is obtained by solving the elementary inverse Cauchy problems (Fig. 2b) from the initial point A. At the points 1 and 2 on the slip line η the values of σ and φ are known, while point 2 belongs to the boundary AB, which is directed along the second principal stress. Therefore, at the points 2 and P of the stress-free boundary $\sigma = -{}^1\!/\!_2$. From Eqs (1.2) for η line 1–2 and for ξ line 1–P we find φ at the point P

$$\varphi = 2\varphi_1 - \varphi_2 \tag{2.4}$$

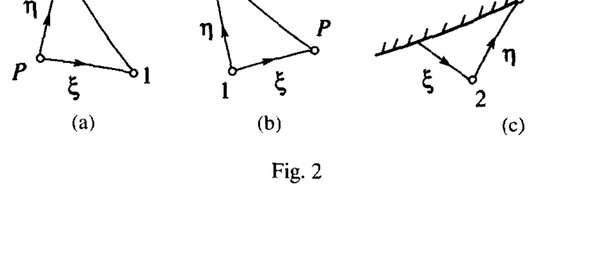

Fig. 2

From the differential equation of the boundary AB $dy/dx = \mathrm{tg}(\varphi - \pi/4)$ and the differential equation of the ξ slip line (1.1), assuming the mean slopes of the tangents $\langle\varphi_1\rangle = \frac{1}{2}(\varphi_1 + \varphi)$ and $\langle\varphi_2\rangle = \frac{1}{2}(\varphi_2 + \varphi) - \pi/4$ between the points $1 - P$ and $2 - P$, we find the coordinates of the point p

$$x = [y_1 - y_2 - x_1 \mathrm{tg}\langle\varphi_1\rangle + x_2 \mathrm{tg}\langle\varphi_2\rangle]/(\mathrm{tg}\langle\varphi_2\rangle - \mathrm{tg}\langle\varphi_1\rangle), \quad y = y_2 + (x - x_2)\mathrm{tg}\langle\varphi_2\rangle \tag{2.5}$$

The velocities V_ξ and V_η on the boundary OA are found by solving the elementary mixed problems (Fig. 2c) for the differential relation (1.3) on the η line $2 - P$ and for boundary conditions (1.10) and (1.11)

$$V_\eta = [V_{\eta 2} - \tfrac{1}{2} V_{\xi 2}(\varphi - \varphi_2)]/[1 + \tfrac{1}{2} a(\varphi - \varphi_2)], \quad V_\xi = aV_\eta \tag{2.6}$$

where for a smooth cylinder $a = 1$, for a rough cylinder $a = \mathrm{tg}\theta$ with forward slipping and $a = \mathrm{ctg}\theta$ with backward slipping; subscripts 2 in (2.6) denote the known velocities and the angle φ at point 2.

Algorithms presented in earlier work [9, 10] for solving the basic boundary-value problems for hyperbolic equations of ideal plasticity are used here with the inclusion of computing procedures for solving the elementary boundary-value problems containing Eqs (2.1)–(2.6). These equations do not require iterations, owing to the linearity of differential equations (1.2) and (1.3), and therefore the calculation of a very detailed network of slip lines and of the displacement velocity field for a given distribution of σ at 20 nodes on the boundary OA is achieved in a fraction of a second.

Let the vector $\boldsymbol{\sigma}$ denote the unknown values of σ at N nodes on the boundary OA, and let the vector $\mathbf{f}$ denote the differences between the slopes of the tangent to the boundary AB and the velocity vector at N nodes on that boundary, representing the errors of the condition of stationarity (1.4) for given $\boldsymbol{\sigma}$. The algorithm for calculating the slip line field and the displacement velocity field determines the continuous dependence of $\mathbf{f}$ on $\boldsymbol{\sigma}$, and the operator equation (1.21) takes the form of a non-linear vector equation of dimension N

$$\mathbf{f}(\boldsymbol{\sigma}) = 0 \tag{2.7}$$

Equation (2.7) is solved using Broyden's method [8], which does not require the calculation of derivatives in the iteration process. The initial approximation of σ^0 was specified by the linear distribution of σ on OA from the known value at the point O (1.15)–(1.17) to the value (1.18) at the point A, assuming the initial approximation for the angle $\beta = c\alpha_c$, where $c > 1$. The functional matrix $\partial f_i/\sigma_j$ at the initial point $\boldsymbol{\sigma}^0$ was found using the finite-difference method, solving N problems for variations of $\boldsymbol{\sigma}^0$.

During the rolling and slipping of a smooth cylinder with small contact angles α_c, the slip line field (Fig. 1) approximates to Prandtl's asymmetrical field for a smooth flat punch with constant distribution of σ on OA and constant velocity $V_x = -1$ in the plastic region. In this case, Eq. (2.7) is solved in 1–2 iterations with an accuracy of $|f_i|_{\max} \leq 10^{-4}$ ($i = 1, 2, \ldots, N$) with $c = 1$. Increasing the parameter c as in the arc of contact increased ensured that Eq. (2.7) was solved with the same accuracy with a small number of iterations. In this case, the condition $y = 0$ at point B is satisfied with an accuracy of 10^{-6}.

3. NUMERICAL RESULTS

To solve the problem, a program was written that calculates the slip line fields and velocity hodographs for a given contact angle α_c of a cylinder with a plastic region and for a given parameter of contact τ_c with forward and backward slipping.

Figure 3 shows a slip line field with the contact pressure distribution (a) and a hodograph of the displacement velocities (b) during the rolling and slipping of a smooth cylinder with a contact angle $\alpha_c = 0.4$. For this version the values obtained are

$$\beta = 0.676, \quad \psi = 0.495, \quad Q = 2.049, \quad F = 0.380$$

For a smooth cylinder, the boundary conditions for the stresses and displacement velocities do not depend on the rotation of the cylinder, and the solutions obtained hold for different values of ω with forward and backward slipping. As the contact angle α_c increases, the angle of the centred fan ψ tends to zero, the region AED on the physical plane contracts to a line, the velocity at the singular point A becomes unique, and the corresponding curve A–A on the displacement velocity hodograph tends towards the stationary point O. Thus, as $\psi \to 0$ the displacement velocity of a material particle along the stress-free boundary AB decreases from unity at point B to zero at point A, and it then increases from zero to unity when the particle moves along the boundary of the cylinder from the point A to the point O. A limiting value of $\alpha_c^* = 0.453$ was obtained for a smooth cylinder.

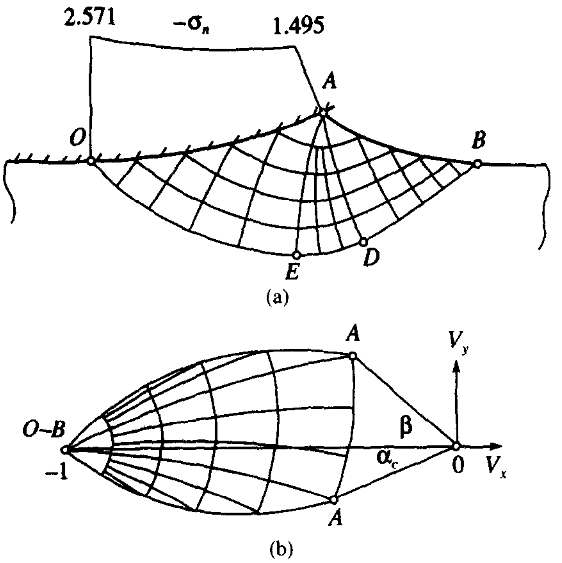

Fig. 3

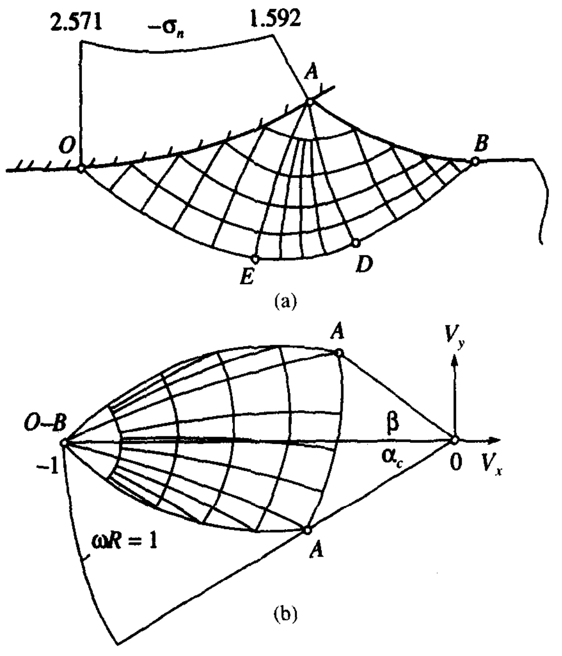

Fig. 4

The values obtained for the rolling of a smooth cylinder with a contact angle $\alpha_c = 0.2$ are

$$\beta = 0.245, \quad \psi = 1.126, \quad Q = 2.342, \quad F = 0.228$$

with a variation of the contact pressure from 2.126 at the point A to 2.571 at point O. For angles $\alpha_c \leqslant 0.2$ the slip line field approximates to a Prandtl field for a flat punch, and the results obtained are identical with the solution of the problem using the small parameter method [3]. For the rolling of a smooth cylinder with small contact angles $\alpha_c < 0.2$ we have

$$Q \approx \tfrac{1}{2}(\pi - \alpha_c), \quad F \approx \tfrac{1}{2}\alpha_c Q; \quad F/Q \approx \tfrac{1}{2}\alpha_c$$

Figure 4 shows a slip line field with the contact pressure distribution (a) and a hodograph of the displacement velocities (b) for the rolling of a rough cylinder without slipping at the point O ($\omega R = 1$)

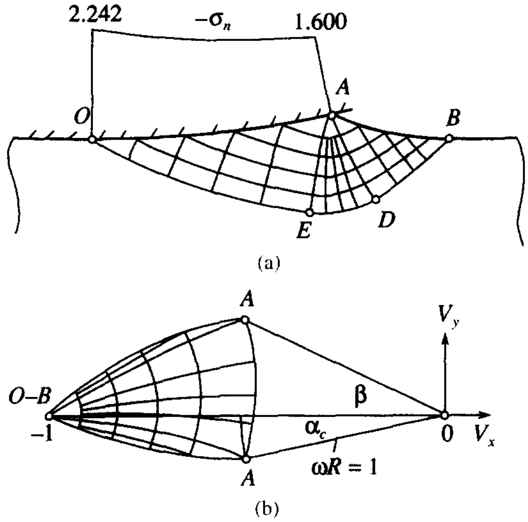

Fig. 5

with contact angle $\alpha_c = 0.5236$ and linear distribution of τ_c with maximum value of 0.25 at the point A. This is a version of the problem with forward slipping, for which the values obtained are

$$\beta = 0.650, \quad \psi = 0.658, \quad Q = 2.070, \quad F = 0.382, \quad M = 0.239$$

An increase in friction during forward slipping leads to an increase in the angle ψ and to an increase in the limiting contact angle α_c^* as $\psi \to 0$.

The ratio F/Q can be treated as the coefficient of rolling friction induced by the asymmetry of the plastic region relative to the y axis passing through the axis of the cylinder. Below we give values of α_c^* for the rolling of a cylinder with forward slipping as a function of the contact friction stress τ_c at the point A, and the integral characteristics of the process Q and F and the ratio F/Q

τ_c	0.0	0.1	0.2	0.3	0.4
α_c^*	0.453	0.552	0.650	0.750	0.875
Q	1.951	1.885	1.828	1.774	1.692
F	0.400	0.416	0.427	0.435	0.440
F/Q	0.205	0.220	0.233	0.245	0.260

In the case of forward slipping for given values of τ_c, the contact angles α_c can be found for which the force F equals zero. This is the limiting case for the rolling of thick workpieces without penetration of plastic strains through the thickness of the workpiece and with plastic deformation of the surface layer only.

Figure 5 shows a slip line field with the contact pressure distribution (a) and a hodograph of the displacement velocities (b) for the rolling of a rough cylinder with backward slipping ($0 \leqslant \omega R < 0.53$) and with contact angle $\alpha_c = 0.2$ and constant contact friction stress $\tau_c = 0.25$. The values obtained for this version are

$$\beta = 0.44, \quad \psi = 0.667, \quad Q = 1.922, \quad F = 0.433, \quad M = -1.25$$

An increase in friction during backward slipping leads to a decrease in ψ, in the limiting contact angle α_c^*, and in the normal pressure on the cylinder, with a more even distribution of the latter compared with a smooth cylinder and with the rolling of a rough cylinder with forward slipping.

When $\omega = 0$ we obtain the slipping of a circular punch with a stationary plastic region formed in front of it, which depends on the vertical force Q and on the contact friction τ_c. This problem also describes the drawing of a thick rod through circular dies with plastic deformation of the surface layer only. When $\tau_c \to 1/2$ with backward slipping $\alpha_c \to 0$, the plastic region $ABDE$ degenerates to the point A, and the region OAE degenerates to the shear line with uniform pressure $\sigma_n = 1/2 + \pi/4$ on the contact line OA. This is the case of the slipping of an absolutely rough flat punch along the boundary of a plastic half-space.

For practical applications, a version of the program has also been written that calculates the streamlines and the non-uniformity of the distribution of the accumulated plastic strain in the plastic region and over the thickness of the deformed layer beyond the cylinder.

Calculations of the rolling and slipping of a cylinder taking into account the contact friction τ_c show the considerable influence of this parameter on the shape of the plastic region and on the forces and torque acting on the cylinder. For small contact angles and with forward slipping increase in τ_c leads to a decrease in the horizontal force F and in the ratio F/Q. A computer program enables the values of α_c and τ_c to be found for which $F = 0$. These are the limiting conditions for rolling thick workpieces with free ends. When τ_c is increased further, the force F increases, but in the opposite direction. For the limiting conditions of the rolling of thick workpieces, a change in the direction of the force F results in a change in the direction of the external "tension" applied to the workpiece, which, with an appropriate value of the vertical force Q applied to the roll, leads to stationary plastic flow of the surface layer.

The model of backward slipping is of practical interest in the technology of the surface plastic deformation of thick workpieces using a slipping, curved (in this case circular) tool, taking into account the contact friction. An increase in τ_c leads to an increase in the force F and the ratio F/Q. In this case, the computer program enables the variation in the forces F and Q and in the shape of the plastic region to be analysed until it degenerates into the shear line of the surface layer.

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